The Fibonacci Project

DISSEMINATING INQUIRY-BASED SCIENCE AND MATHEMATICS EDUCATION IN EUROPE

TOWARD NEW TEACHING IN MATHEMATICS

Conclusions from the SINUS and SINUS-Transfer Pilot Studies

WITH THE SUPPORT OF

EUROPEAN COMMISION

SEVENTH FRAMEWORK PROGRAMME

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> "Everything has been said before, but since nobody listens we have to keep going back and beginning all over again."
>
> *A. Gide*

Defining where we stand

International comparative studies such as TIMSS and PISA not only provide certain information about teaching, but are indisputably necessary and meaningful. However, the aforementioned tests and the resultant rankings should not be accorded greater significance than the purpose and content areas of teaching and thus of education itself. This concern also applies to the grade level tests that have meanwhile become obligatory in an increasing number of federal states in Germany. Instruction should not deteriorate into "teaching to the test." Unfortunately, undesirable developments in this direction can already be identified.

Teaching will not improve simply by being evaluated and empirically investigated. In order to actually eliminate the recognized deficiencies, concrete, content-related concepts are required in order to change instruction and learning. General recommendations and requirements without reference to concrete subject-related contents do not result in change. Instructors are particularly likely to accept and implement suggestions in a positive spirit when they are offered in the form of sound technical and didactic learning environments.

For many years, constructive methodological development research has been carried out, with the design of learning environments as well as their empirical research, testing and implementation forming the focus of such work. The BLK*) pilot studies Increasing the Efficiency of Mathematics and Science Instruction SINUS and SINUS-Transfer profited from this constructive development work. In my view, ensuring that these results are incorporated into both the schools and into initial and advanced teacher training represents one of the most urgent educational policy tasks, regardless of any reform of Germany's federal system. The Fibonacci Project – to be funded by the European Union as of 2010 and based on SINUS with regard to the sphere of mathematics – offers a new opportunity to further enhance teaching at German schools and one that should be seized with alacrity.

*) BLK – Bund-Länder Commission for Educational Planning and Research Promotion (until 2007)

Starting points for changes in teaching and learning

1. Guide to independent work

> "A student is mature when he has learned enough to be in a position to learn for himself."
>
> *W. v. Humboldt*

Nowadays most students do not acquire this kind of autonomy. Teachers serve up the subject matter, students promptly forget it or never absorb it in the first place. ("Knowledge is served, swallowed, forgotten.") How can we escape this dilemma? The only way out is to change the roles: The teacher is not an entertainer, and the student is not a mere consumer. Therefore no ready-made mathematics should be offered. Teaching must not be limited to a teacher, for example, demonstrating a method of calculation and students subsequently repeating it without reflection. Learning is an active, constructive, cumulative and goal-oriented process; this is something that must also be perceptible for the students.
For this reason, the concept of "subject-related learning" should be expanded upon. Much more is involved than simply providing basic knowledge. Together with this, i.e. on the basis of concrete subject matter, we must at the same time develop the student's ability to engage in active, independent learning.

2. Not only basic knoledge, but education

Of course solid basic knowledge provides the necessary foundation for successful continued learning. But schools must provide for more than mere knowledge, they must provide education. Yet what does that mean? Education means, among other things, to be able to deal with knowledge, to be able to apply knowledge, to be able to make valuations. These abilities are of a different quality than simply checking off isolated elements of knowledge. Independent work and thinking as well as communication and teamwork are required. In this manner, the respective education standards automatically find their way into instruction.

Mathematical education consists, among other things, of the following elements:

- appreciation of mathematics
- confidence in one's own ability to be able to deal with mathematics
- application of mathematical knowledge to mathematical and non-mathematical tasks
- the ability to use mathematics as a form of communication and expression
- mathematical thinking.

3.Guidelines for teaching mathematics

Even SINUS does not provide one all-purpose solution for successfully teaching mathematics. Many paths lead to this goal, some of them quite different from one another. Successful mathematics teachers tend to be markedly individual. However, there are certain fundamental guiding concepts that typify instruction at SINUS schools.

These include

- less knowledge acquisition, more problem-solving instruction,
- less orientation to calculation, more focus on comprehension,
- learning mathematics in context ("storytelling", mathematics as a cultural asset),
- attention not only to results but also to the learning strategies and learning processes required. The journey is the reward!

Implementation of these central concepts leads away from rigid mathematical instruction in its traditional form, characterized by a narrowly conceived question-and-answer approach with formal arithmetic and the manipulation of terms in the forefront of interest.
We must also consider the fact that the receptiveness of many students with regard to pure subject matter has substantially declined under the influence of everyday media consumption. This does not automatically mean that today's students are worse than those of former times; however, they are different and thus they behave differently. So we do not need new content but rather a different approach to dealing with old content. We have to take this into consideration when it comes to teaching.

4. Experimental access to mathematics: finding independent paths to learning

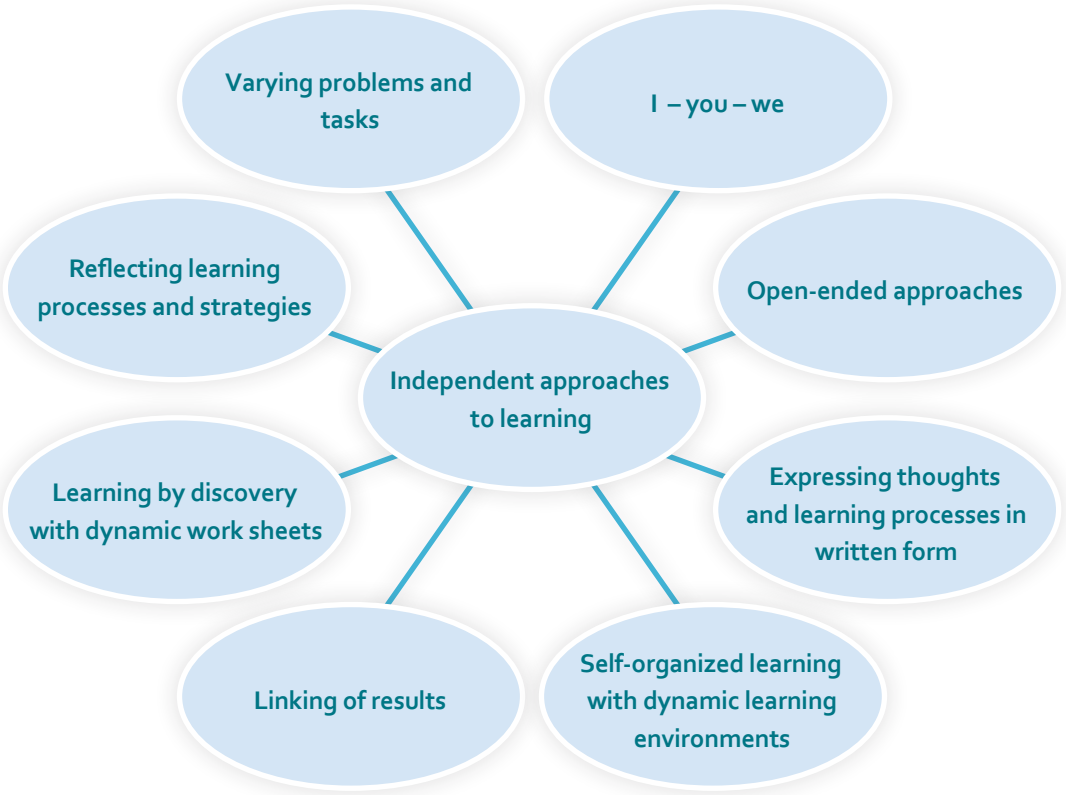
What is the idea behind the call for classroom innovations? Our schools have focused far too long on the transfer of knowledge. Students usually remain passive, while the teacher struggles to convey the relevant material all on his/her own. The compulsive notion that "I have to get through my material" affords less latitude for independent contributions from students. It is for this reason that we need a fundamental reorientation. The priority among instructors should not be to pass on their knowledge to students, but rather to make it possible for students to access knowledge on their own instead.
Many years ago the American mathematician Paul Halmos (1916 – 2006) demanded: "Don't preach facts, stimulate acts." Applied to school, this means that students should be induced and encouraged to explore their own learning resources. They should

- experiment,
- observe
- discover
- conjecture
- explain and justify.

This approach describes exactly how experimental work operates within the scope of mathematical instruction. In the beginning there are never rules or formulas; these are derived, if at all, at the end of the learning process.
Another thing sustainable instruction requires is that we create situations in which students (can) develop interest and then go through learning processes in the following stages:

- alone (I),
- with other students (you),
- with the help of the teacher (we).

This brings us to one of the main focuses of the work of SINUS and/or SINUS-Transfer when it comes to the subject of mathematics. The underlying methods can be transferred to other subjects accordingly.



Independent knowledge construction does not by any means preclude the systematic transfer of knowledge and instructional support. It is only the interaction of all of these forms of instruction that paves the way for effective and sustainable learning processes.

5. Innovations in the classroom – "Lesson Study Process"

If there is no change in the type of teaching, then better results will not be obtained, no matter how much the material handled or the size of the respective classes are reduced. Well-established and practiced teaching experience steeped in tradition cannot be changed so easily – and certainly not outside the realms of the school (university, parents' associations, and industry) or by means of ministerial decrees. It is a truism that a lesson or teaching unit is determined in terms of both content and method by the personality of the instructor. Thus a change of the teaching script can only take place through the teacher.

Greater changes are possible through many small changes over a longer period. We need a process of continuous improvement in teaching that is designed by the teachers themselves, comparable with the successfully practiced "lesson study process" in Japan. In order to initiate and implement such a process, teachers require competent support and outside consultation, which should never be patronizing.

Along the lines of management consulting, we imagine the establishment of advisory offices for classroom innovations to which teaching staff may turn to obtain information about the latest

developments in teaching methodology, pedagogy and educational psychology. This also includes competent support in the selection of teaching materials and instructional software. These advisory offices could, for example, be established within the (existing and planned) university centers for teacher training and school research (cf. Center of Mathematics and Science education at the University of Bayreuth).

6. Good practice networks

We select good practice schools – for example, from the SINUS and/or SINUS-Transfer school sets, as well as from the network of MINT-EC schools – and document what is done differently in these schools and how this happens. These facilities serve as pilot schools for the establishment of new networks (e.g. for the Fibonacci Project), while organization and structure are oriented toward SINUS. In the next step it is important to encourage cooperation among the teachers in the network, e.g. the exchange of experience and materials, shared advanced training, joint development of learning environments, mutual attendance of lessons. The central SINUS server (http://sinus-transfer.eu) can be employed for these activities.
Networks can be formed even without a school networking structure. There are enough interested and committed instructors. We only have to create opportunities for them to have regular meetings and exchange experience.

7. Involving participants and public

It is clear to anyone concerned with education that innovative curricula, educational standards and proven learning environments offer no guarantees for positive changes in teaching. As has already been noted, the quality of instruction stands and falls with the person of the teacher. Instruction depends on the professionalism and motivation of the instructors themselves. Effective changes are possible – as is proven by the SINUS pilot study. The basic prerequisite, however, consists in successfully convincing the participating instructors of the relevant ideas and actively involving them in the processes of change.
However, the commitment of the instructors alone is not enough to change the teaching of mathematics. Certain framework conditions must also be fulfilled. A competitive school will not succeed without the support of society. This means

- that the school administration and supervisory authorities must expressly support and promote the required reforms;
- students and parents must accept the fact that there cannot be scholastic achievement without effort and dedication.

8. Introducing to the "SINUS Philosophy"

It has already been stated that successful instruction has an individual face that is primarily that of the individual instructors. Ideas and materials from SINUS provide inspiration, but implementation always has a personal touch.
What distinguishes successful mathematics instruction at SINUS schools? How can the small steps specified above be given a concrete form that will lead to a new form of instruction? Access to the

"SINUS philosophy" is best achieved through conscious consideration of one's own teaching. Here certain central themes can serve as a means of orientation. These themes take five different aspects of teaching into consideration (explanations provided in the Annex):

1. Teaching style
2. Work with problems/tasks
3. Technical contents
4. Type of achievement testing
5. The role of mathematics teachers.

Reflection based on these central themes also makes sense for education degree students and probationary teachers. Although first-time instructors usually have only very limited teaching experience (if any), these central themes clearly point to crucial areas for subsequent instructional activity.
A first step is taken by reconsidering one's own instruction on the basis of the central themes indicated. Thus the foundation is laid for a change in teaching. What remains then is for implementation to take place in accordance with the slogan cited from Paul Halmos: "Don't preach facts, stimulate acts."

Annex

Central Themes - Reconsidering One's Own Teaching

1. Reconsider your teaching style

- You do not represent the focus of instruction, your students do.
- Support your students in their learning, avoid lecturing them.
- Encourage your students to explore their own paths to learning.
- Provide suggestions and assistance towards self-help.
- Clearly separate learning and testing situations.
- Vary forms and methods of classroom teaching.
- In the final analysis it is not you but your students who are responsible for their learning progress.

2. Reconsider your work with problems

- Do not have your students simply produce answers, make them get involved with the respective questions. The journey is the reward.
- Make it possible to actively and productively work with problems:
  - initiate problems and tasks;
  - vary problems;
  - recognize patterns;
  - prepare strategies for solutions.
- Find different paths toward solutions and then take them.
- Link everyday knowledge and mathematical knowledge with one another in a meaningful way.
- Have students keep study journals. Teach them how to express problem-solving and learning processes in written form.

3. Reconsider the subject matter

- Limit yourself to the fundamental content.
- Emphasize the essential ideas of the respective subject matter.
- Treat the contents in an appropriate and interesting context.
- Attach value to discovering and working out relationships involving content and structure.
- Reduce the prevailing orientation toward calculation in favor of a focus on comprehension.

4. Reconsider your conventional manner of testing

- Does testing always have to involve calculation?
- Can a test item take the form of a description?
- Can explanations and justifications be built into a "traditional problem"?
- Can problems be prepared in such a way that various solutions are possible and make sense?
- Variation of test items can be more demanding than formal application of a calculation method.
- Evaluate how study journals are maintained.
- Include checks of learning objectives in planning your teaching units.

5. Reconsider your own role as a mathematics teacher

- Express your enthusiasm for mathematics.
- Repeatedly emphasize the importance of mathematics, particularly in terms of culture, technology and industry.
- Show personal interest in the subject matter that you teach.
- Continue to be actively involved in mathematics: problem-solving, competitions, popular science literature, etc.
- Avoid being a "lone wolf", rely on cooperation among the student body and/or teaching staff.
- Through your teaching show that mathematics is a vibrant, constantly developing discipline.

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