

Disorder-driven multifractality transition in Weyl nodal loops^[1]

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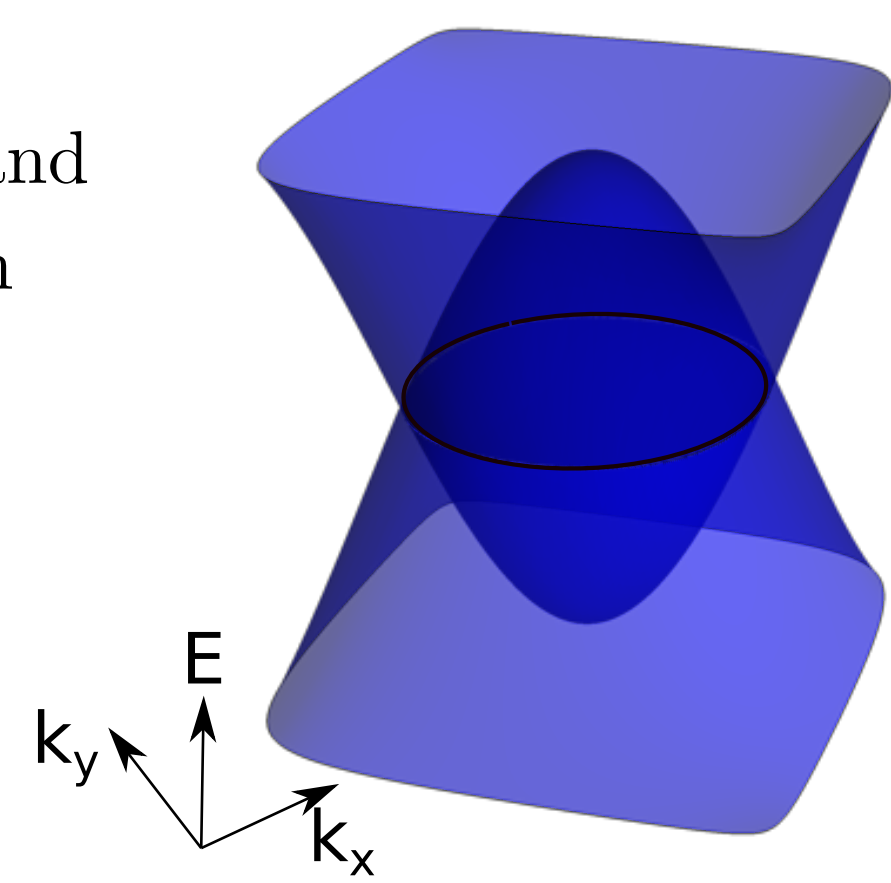
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Weyl nodal loop (WNL) + short-range disorder

Clean limit [2]

- 3D topological semimetal where the conduction and valence bands touch along 1D loops in momentum space;
- Linear low-energy dispersion relation;
- Linearly vanishing DOS at the Fermi level;
- Drumhead surface states.



Model for WNL + short-range disorder

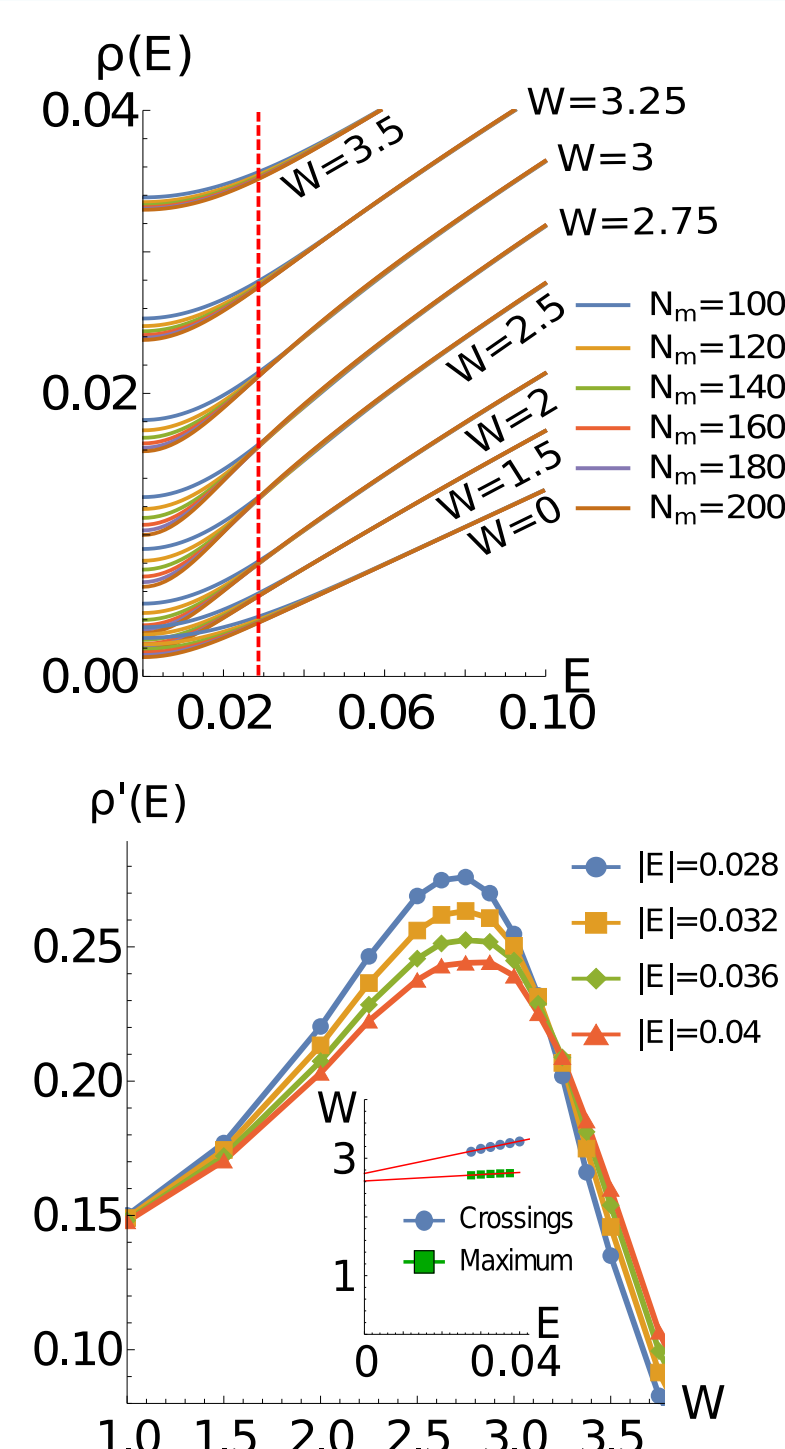
- Clean limit modeled by simple two-band model with nodal-loop for $k_z=0$;
- Disorder modeled by uniform box potential of width W . The random potential is uncorrelated for different sublattices.

$$H = \sum_k c_k^\dagger H_k c_k + \sum_r c_r^\dagger V_r(W) c_r \quad \left[\begin{array}{l} V_r(W) = \text{diag}(v_{r1}, v_{r2}) \\ v_{r\alpha} \in [-W/2, W/2] \end{array} \right]$$

$$H_k = (t_x \cos(k_x) + t_y \cos(k_y) + \cos(k_z) - m)\tau_x + t_2 \sin(k_z)\tau_y$$

Semimetal – metal transition

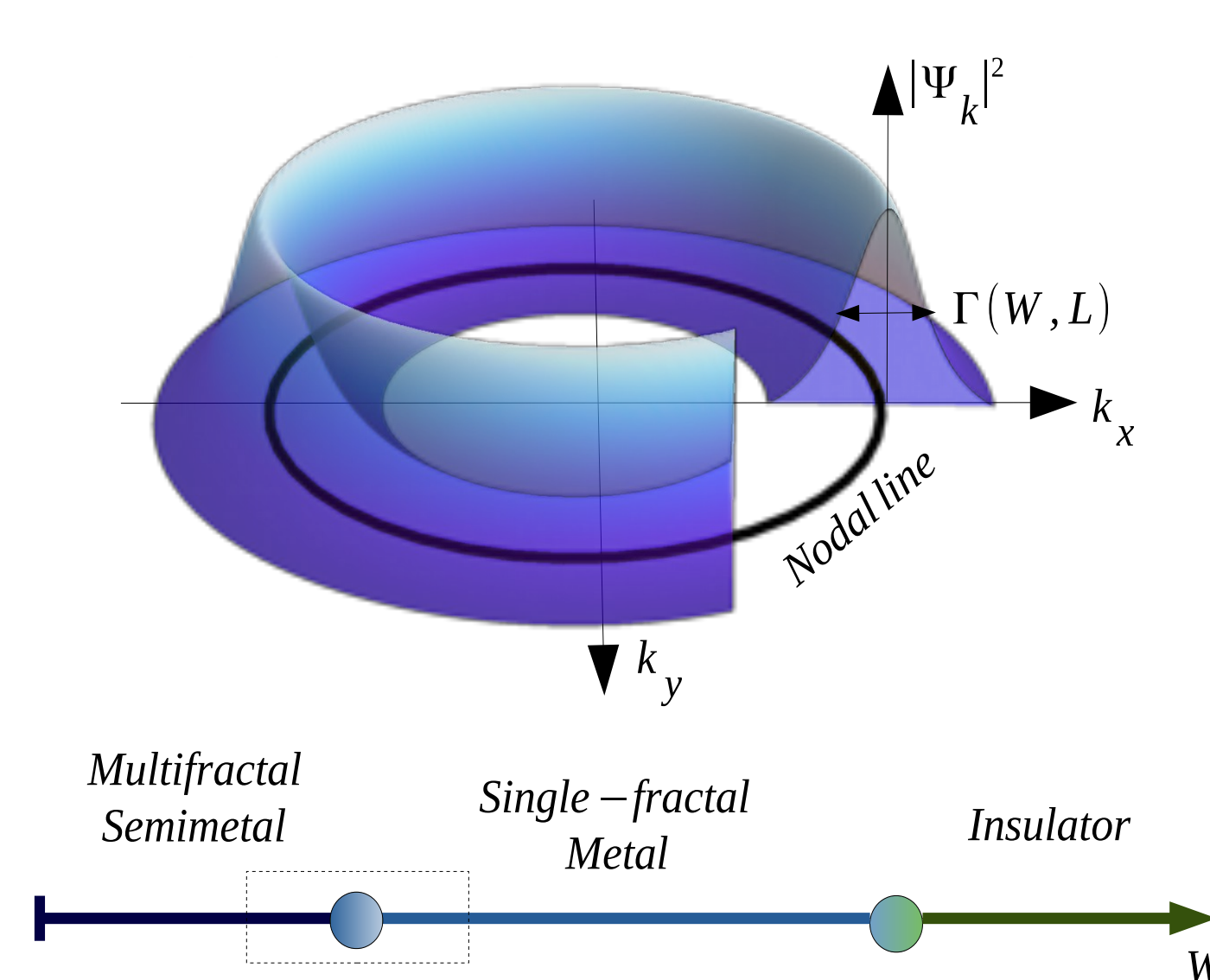
- DOS computed with Kernel Polynomial Method [4];
- Close to $E=0$, the DOS strongly depends on the number of Chebyshev moments N_m :
 - $\rho(E)$ does not converge inside an energy window around $E=0$;
 - We trust the results only outside the non-converged window;
- For a finite $\rho(E=0)$ we expect $\rho'(E) \rightarrow 0$ as $E \rightarrow 0$:
 - Only observed for $W > W_c = 2.6 \pm 0.1$.



Conclusions

- A clean WNL is unstable to an infinitesimal amount of disorder and flows to a strong-coupling fixed point, a novel phase - here dubbed multifractal-semimetal;
- The multifractal-semimetal exhibits momentum-space multifractality and a vanishing DOS at the Fermi level;
- Upon increasing disorder, a transition between the multifractal-semimetal and a single-fractal metal takes place, characterized by the critical exponents $Z=1.9 \pm 0.1$ and $\nu=1.0 \pm 0.2$.

Phase diagram and multifractality transition



• Multifractal semimetal (MF-SM):

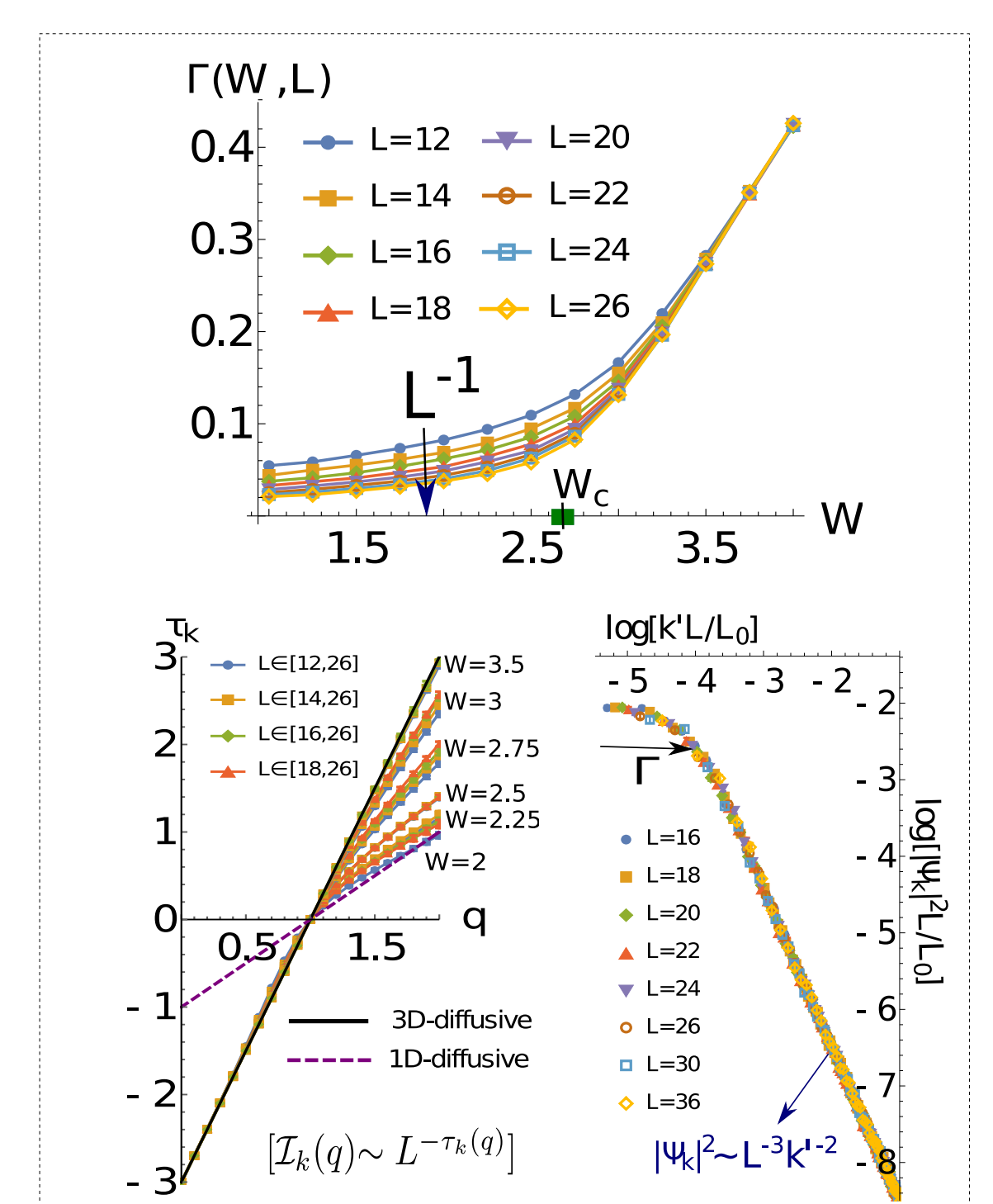
- Nonlinear behaviour of $\tau_k(q)$;
- Vanishing DOS at $E=0$.

• Single-fractal metal (SF-M):

- Linear behaviour of $\tau_k(q) = \beta(q-1)$;
- Finite DOS at $E=0$.
- MF-SF and SM-M critical points match, up to numerical accuracy.

• Anderson insulator:

- Real-space localized eigenstates at $E=0$.



Generalized momentum-space inverse participation ratio [3]:

$$\mathcal{I}_k(q) = \sum_{\alpha} |\Psi_{k,\alpha}|^{2q} \propto L^{-\tau_k(q)}$$

Width of the wavefunction around the nodal-loop: $[\mathcal{I}_k \equiv \mathcal{I}_k(q=2)]$

$$\Gamma = \frac{2\pi}{\sqrt{\mathcal{I}_k} L^3} \sim \begin{cases} L^{-1}, & \text{MF phase} \\ L^0, & \text{SF phase} \end{cases}$$

Scaling analysis

Scaling variables:

$$\Gamma = f_s(L/\xi_s) \quad [W < W_c]$$

$$\Gamma^{-1}/L = f_m(L/\xi_m) \quad [W > W_c]$$

$$\xi_s, \xi_m \sim \delta^{-\nu}, \delta = |W - W_c|/W_c$$

Density of states [5]:

$$\rho(E) \sim |E|^{\frac{d}{z}-1}, W = W_c$$

$$\rho(E) \sim \delta^{\nu(d-z)} \mathcal{F}_\gamma(\delta^{-\nu z} |E|)$$

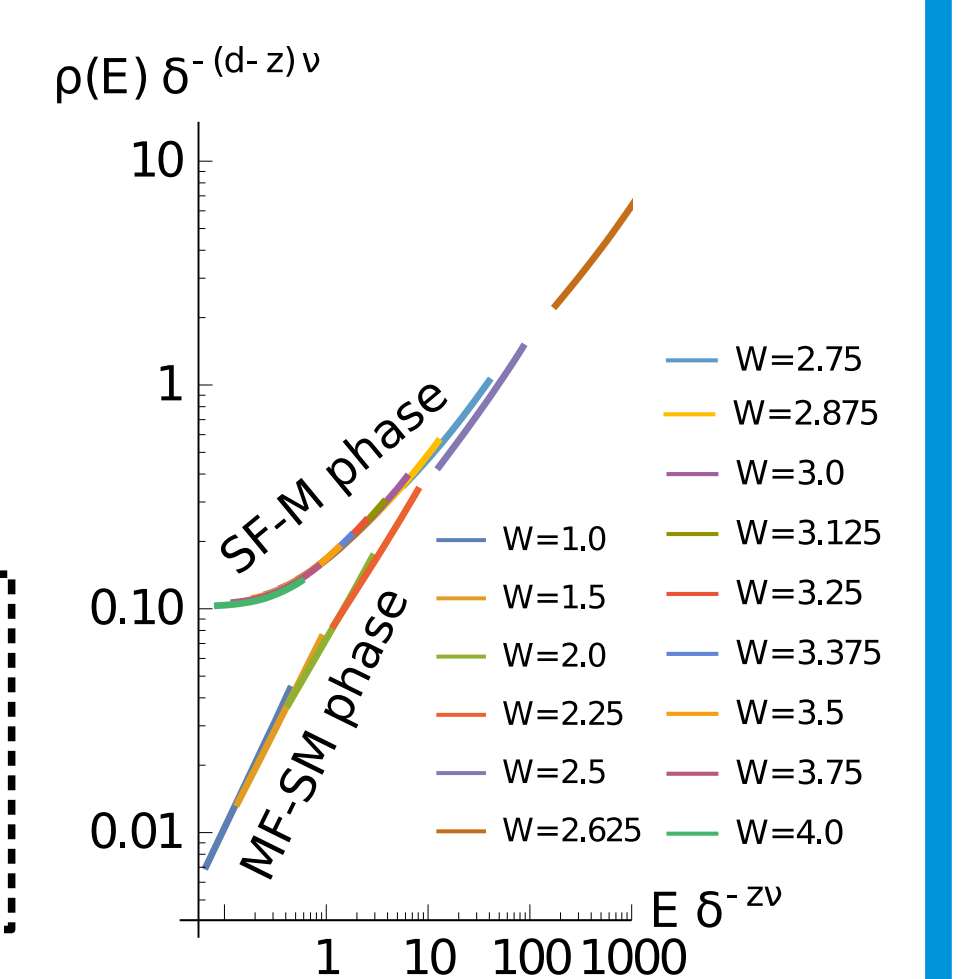
$$Z = 1.9 \pm 0.1$$

$$\nu = 1.0 \pm 0.2$$

+

$$W_c = 2.6 \pm 0.1$$

Average of SM-M and MF-SF critical points



References

- [1] M. Gonçalves, P. Ribeiro, E. V. Castro, and M. A. N. Araújo, Phys. Rev. Lett. 124, 136405 (2020)
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