

Algebraic Geometry of Causal Models

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May 16th, 2022

Encontro Ciência 2022

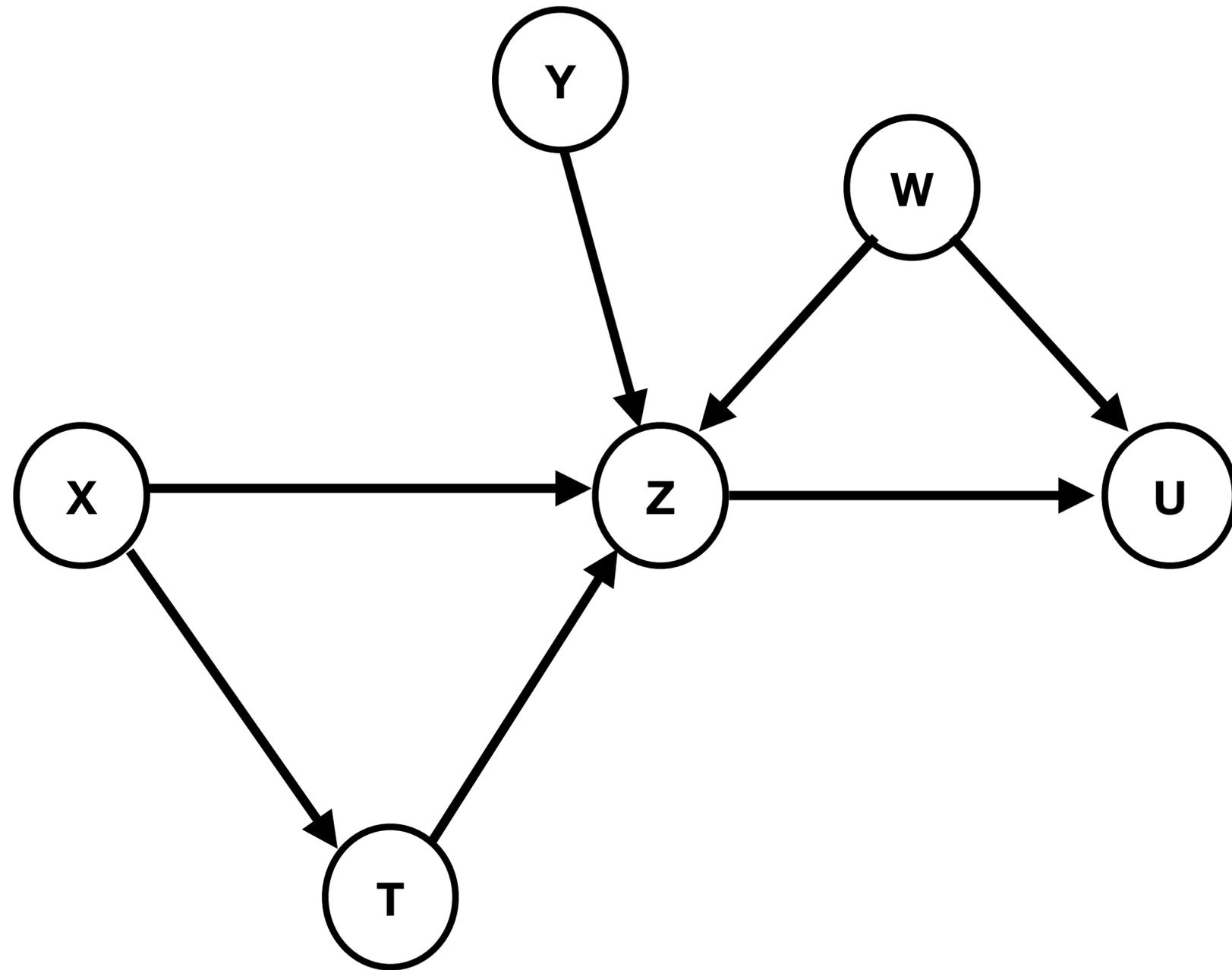


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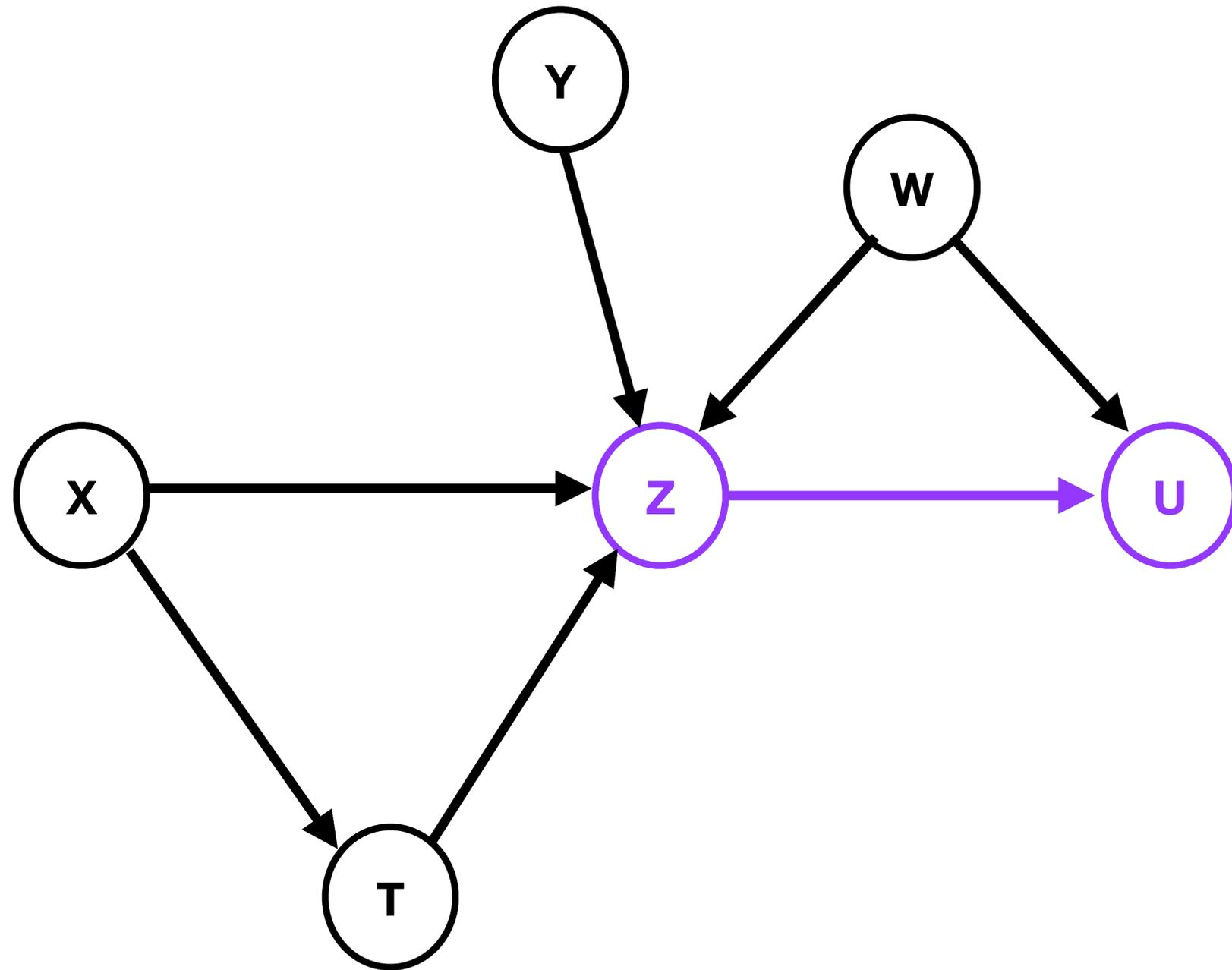


DAG =
Directed
Acyclic
Graph



DAGs represent causal models

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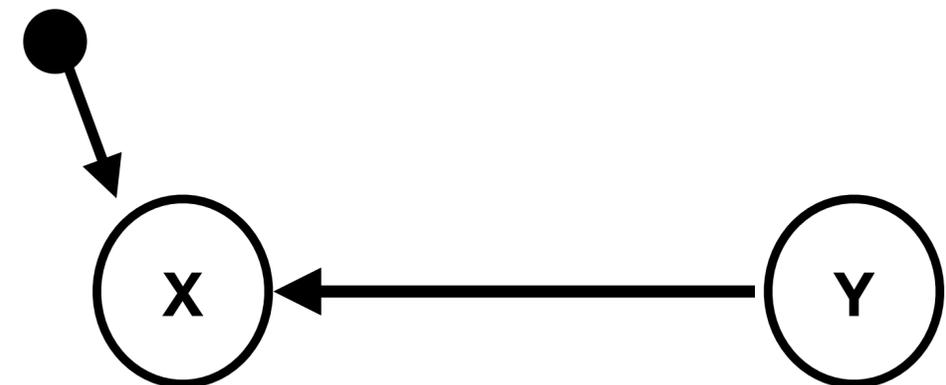
DAGs represent causal models

What is the true causal relation?

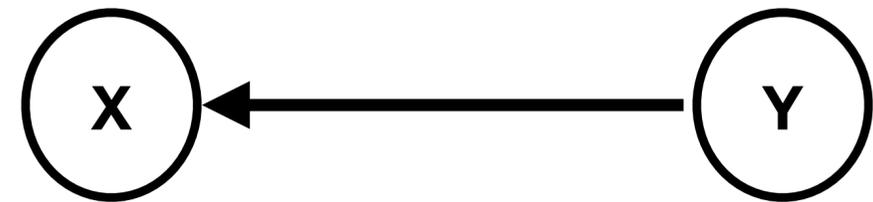
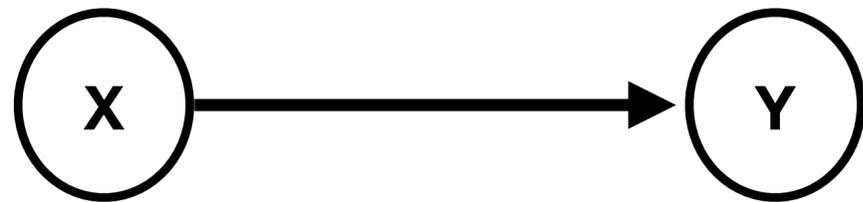


From observational data not possible to decide correct direction

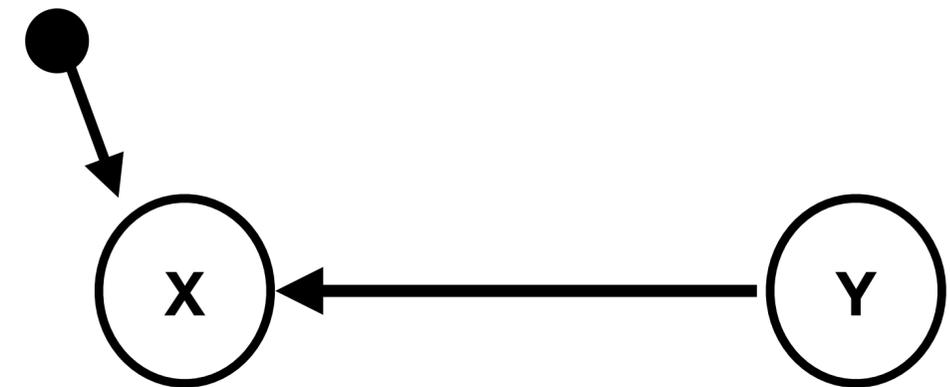
Interventional data: perform experiments that target a subset of the variables



X, Y random variables with outcomes {0,1}



$$\Delta_3^\circ = \{ (p_{00}, p_{01}, p_{10}, p_{11}) \in \mathbb{R}^4 : p_{00} + p_{01} + p_{10} + p_{11} = 1 \}$$

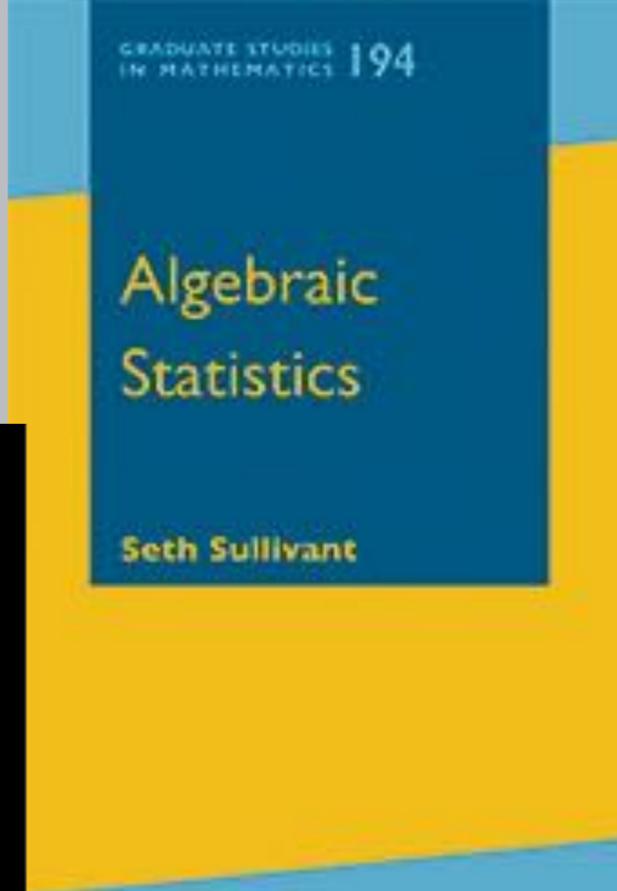
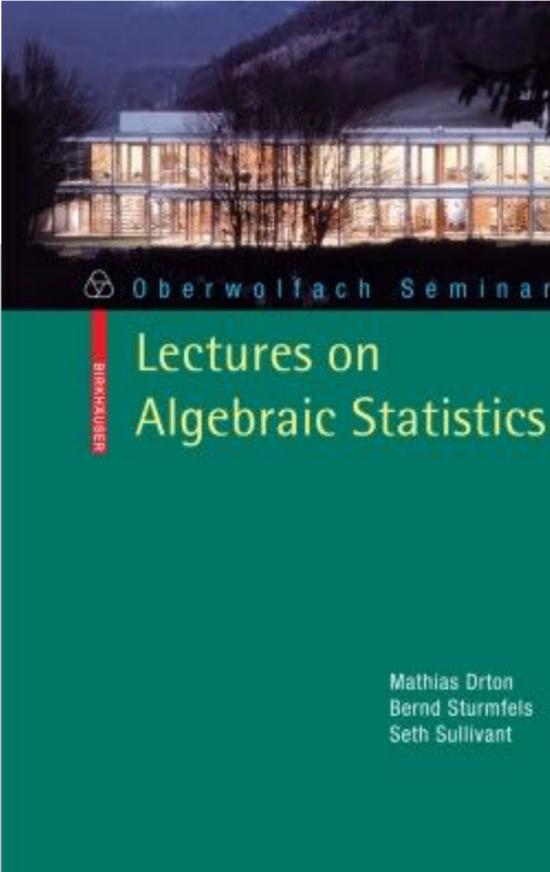
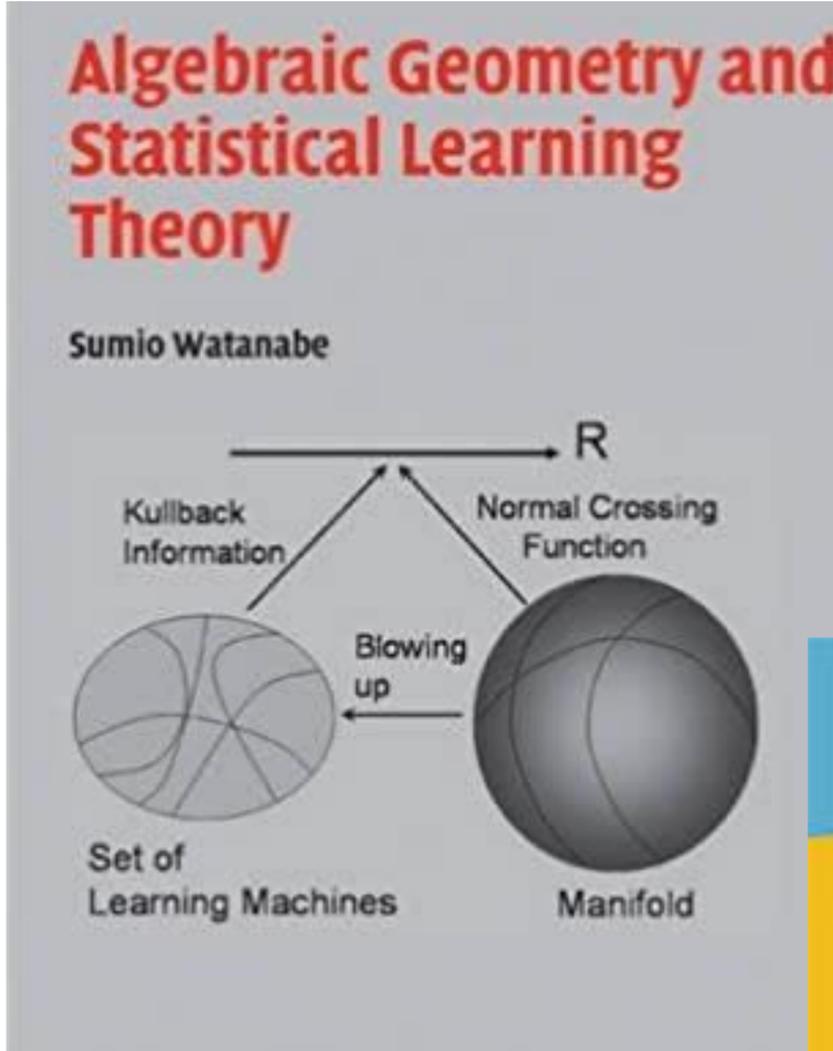


$$\begin{cases} p_{00}^\emptyset p_{01}^I - p_{11}^I p_{10}^\emptyset = 0 \\ p_{10}^\emptyset p_{11}^I - p_{21}^I p_{22}^\emptyset = 0 \end{cases}$$

$$\{ (p_{00}^\emptyset + p_{01}^\emptyset)(p_{11}^I + p_{10}^I) - (p_{00}^I + p_{01}^I)(p_{10}^\emptyset + p_{11}^\emptyset) = 0 \}$$

Algebraic study of causal models

- Models are defined by systems of polynomial equations
- Polynomial constraints are useful to use in causal discovery algorithms and model selection
- Understanding the geometry of the model is useful to study their statistical properties



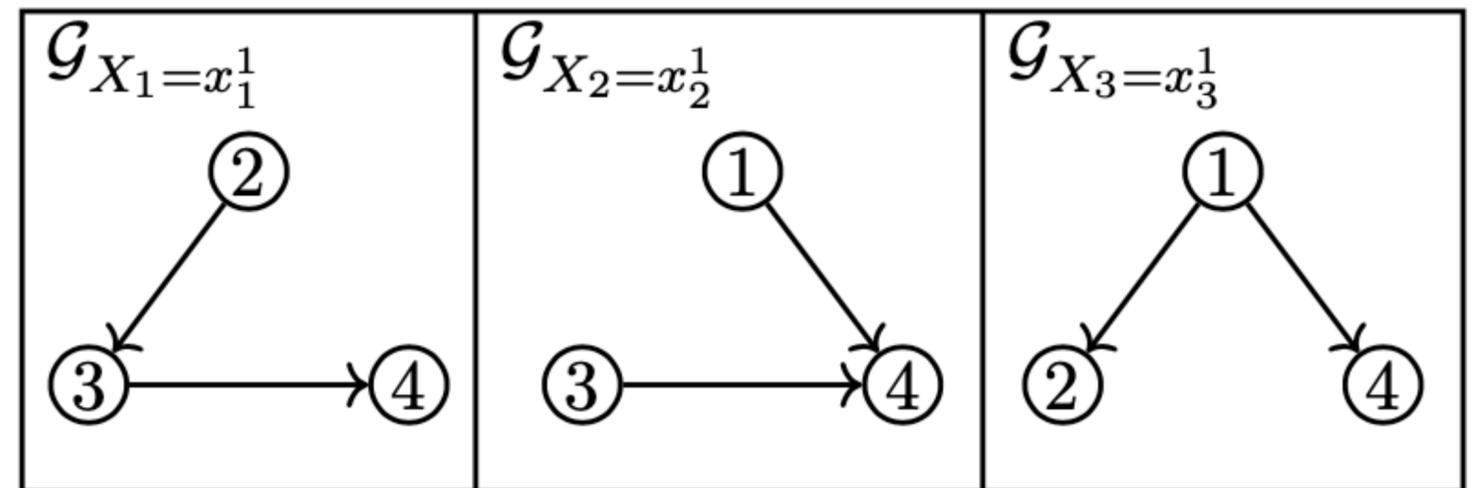
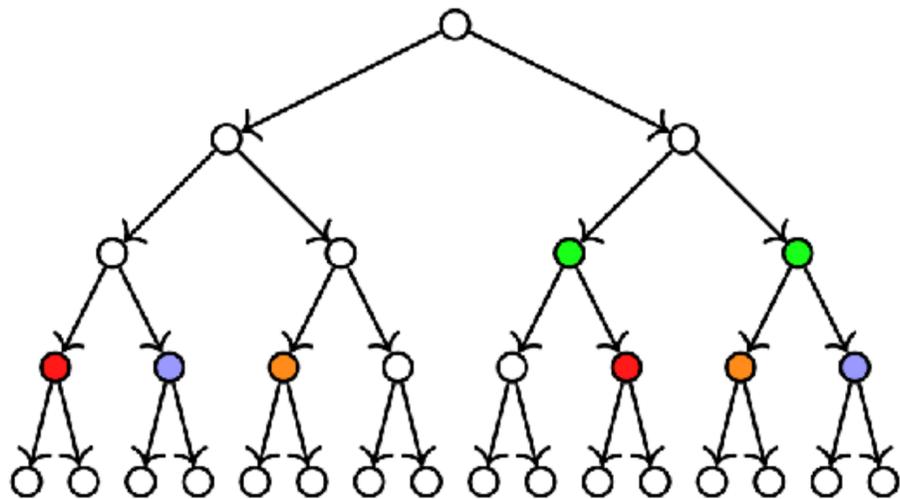
AIToGeLiS
Revealing structures of data with algebra, topology, and geometry.

Algebraic Statistics Online Seminar



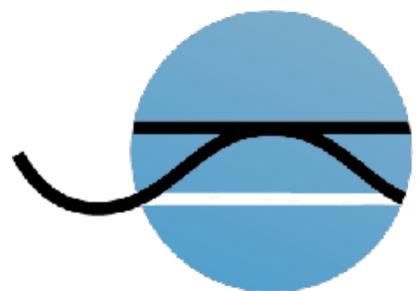
Algebraic study of causal models

- Defining equations of context-specific causal models
- Characterization of discrete interventional DAG models that are toric varieties (i.e. exponential families)
- Generalization to interventions on context-specific causal models



REFERENCES

- [1] Duarte, E. and Görge, C., 2020. Equations defining probability tree models. *Journal of Symbolic Computation*, 99, pp.127-146.
- [2] Duarte, E. and Solus, L., 2020. Algebraic geometry of discrete interventional models. *arXiv preprint arXiv:2012.03593*.
- [3] Duarte, E. and Solus, L., 2021. Representation of Context-Specific Causal Models with Observational and Interventional Data. *arXiv preprint arXiv:2101.09271*.
- [4] Uhler, C., Raskutti, G., Bühlmann, P. and Yu, B., 2013. Geometry of the faithfulness assumption in causal inference. *The Annals of Statistics*, pp.436-463.
- [5] Evans, R.J., 2018. Margins of discrete Bayesian networks. *The Annals of Statistics*, 46(6A), pp.2623-2656.
- [6] Améndola, C., Drton, M., Grosdos, A., Homs, R. and Robeva, E., 2020. Higher Moment Varieties of Non-Gaussian Graphical Models.



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